

Testing spontaneous wave-function collapse models on classical mechanical oscillators

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We show that the heating effect of spontaneous wave-function collapse models implies an experimentally significant increment ΔT_{sp} of equilibrium temperature in a mechanical oscillator. The obtained form ΔT_{sp} is linear in the oscillator's relaxation time τ and independent of the mass. The oscillator can be in a classical thermal state, the effect ΔT_{sp} is classical for a wide range of frequencies and quality factors. We note that the test of ΔT_{sp} does not necessitate quantum state monitoring but tomography. In both gravity-related (DP) and continuous spontaneous localization (CSL) models the strong-effect edge of their parameter range can be challenged in existing experiments on classical oscillators. For the CSL theory, the conjectured highest collapse rate parameter values become immediately constrained by evidences from current experiments on extreme slow-ring-down oscillators.

Spontaneous collapse models [1] suggest that large spatial superpositions of quantum states of massive degrees of freedom, also called Schrödinger Cat states, decay at (model dependent) universal rates. These models, the particular gravity-related (or DP) model [2–5] and the continuous spontaneous localization (CSL) model [6, 7] predict the progressive violation of the quantum mechanical superposition principle for massive degrees of freedom. For atomic degrees of freedom this violation is irrelevant while for massive degrees of freedom it becomes significant though usually masked by the environmental noise. The preparation of Schrödinger Cat states is extremely demanding hence the direct experimental test of spontaneous collapse has not yet been achieved despite relentless efforts, see, e.g., [8–14], and [15, 16] for the state-of-the-art. Quite recently, Bahrami et al. [17] suggested a different approach, not requesting laboratory Schrödinger Cat states. Nimmrichter et al. [18] discuss the optomechanical sensing of spontaneous momentum diffusion caused by collapse models. We further elucidate and simplify these considerations and come to new results. We emphasize that momentum diffusion is classical and this facilitates the mathematical treatment, theoretical insight and experimental proposals. Currently available mechanical oscillators of extreme long ring-down time, e.g.: in the Ref. [19] by Matsumoto et al., are immediately capable of sensing spontaneous heating if it exists with the strongest proposed rates.

Spontaneous collapse models [1] impose spontaneous kinetic energy increase at constant rate proportional to the spontaneous collapse rate. This spontaneous heating is independent of the quantum state which can be a classical state, it need not to be a Schrödinger Cat state for being spontaneously heated.

While spontaneous collapse is a genuine quantum effect, spontaneous heating is not. This we exploit in our work, an elementary (non-quantum) calculation yields the spontaneous increment ΔT_{sp} of the equilibrium temperature T of damped mechanical oscillators. Full quantum calculations can be safely replaced by classical cal-

culations as long as the oscillator remains in the classical domain. Most surprisingly, it turns out that in the classical domain the current laboratory technique is already capable to test the spontaneous collapse models.

Spontaneous heating in oscillators. Let us consider a quantized oscillator of mass m and frequency Ω , with Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\Omega^2\hat{x}^2. \quad (1)$$

If the mass is subject to spontaneous collapse, the density matrix $\hat{\rho}$ satisfies the following master equation:

$$\frac{d\hat{\rho}}{dt} = \frac{-i}{\hbar}[\hat{H}, \hat{\rho}] - \frac{D_{\text{sp}}}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}]] \quad (2)$$

where D_{sp} governs the strength (rate) of spontaneous decoherence. This \hat{x} -decoherence is observable: it is simply equivalent with \hat{p} -diffusion of diffusion constant D_{sp} .

From now on and through our work, we assume that the oscillator is in the classical domain. Therefore we can describe it by the classical Liouville density $\rho(x, p)$ and the quantum master equation (2) can be replaced by the Liouville equation

$$\frac{d\rho}{dt} = \{H, \rho\} + D_{\text{sp}} \frac{\partial^2}{\partial p^2} \rho. \quad (3)$$

$H(x, p)$ is the classical Hamilton function of the oscillator, the Poisson bracket $\{H, \rho\}$ stands for $(p/m)\frac{\partial}{\partial x}\rho - m\Omega^2 x \frac{\partial}{\partial p}\rho$. In a realistic situation, the mechanical oscillator is in a thermal environment of temperature T , which will modify the Liouville equation:

$$\frac{d\rho}{dt} = \{H, \rho\} + D_{\text{sp}} \frac{\partial^2}{\partial p^2} \rho + \eta \frac{\partial}{\partial p} p \rho + D_{\text{th}} \frac{\partial^2}{\partial p^2} \rho, \quad (4)$$

where η is the damping rate of oscillations and $D_{\text{th}} = \eta m k_B T$ is the constant of thermal momentum-diffusion. With $D_{\text{sp}} = 0$ we would get the classical Fokker-Planck equation whose stationary solution is the Gibbs canonical distribution $\mathcal{N} \exp(-H/k_B T)$. It is trivial to see that

with $D_{\text{sp}} > 0$ the stationary solution is the Gibbs canonical distribution

$$\rho_{\infty}(x, p) = \mathcal{N} \exp\left(-\frac{H(x, p)}{k_B T'}\right) \quad (5)$$

at the higher temperature

$$T' = \left(1 + \frac{D_{\text{sp}}}{D_{\text{th}}}\right) T \equiv T + \Delta T_{\text{sp}}. \quad (6)$$

This result can be interpreted as the extension of the Einstein-Smoluchowski relationship $D_{\text{th}} = \eta m k_B T$ for $D_{\text{th}} + D_{\text{sp}} = \eta m k_B T'$, supported by the underlying Fokker-Planck equation.

The increment $\Delta T_{\text{sp}} > 0$ over the environmental temperature T is the contribution of spontaneous heating, this is the very observable quantity that we wish to test. From Eq. (6), we can express it as

$$\Delta T_{\text{sp}} = \frac{D_{\text{sp}}}{m k_B} \tau, \quad (7)$$

where $\tau = 1/\eta$ will stand for the (energy) relaxation time of the oscillator. Our classical description is valid as long as the spontaneous heating concerns many quanta of the oscillator:

$$k_B \Delta T_{\text{sp}} \gg \hbar \Omega. \quad (8)$$

Measurement. Since we restrict ourselves for the classical domain (8) of spontaneous heating ΔT_{sp} , a single-shot classical (or quantum) measurement of precision δT_{m} would detect ΔT_{sp} provided $\delta T_{\text{m}} \lesssim \Delta T_{\text{sp}}$. If this condition does not hold, we repeat the same measurement many times, like in quantum state tomography. Observe that quantum state monitoring is not necessary, tomography is the more suitable means to detect spontaneous temperature increase of the previously prepared equilibrium oscillator state. Cumulative precision of tomography is not limited quantum theoretically.

For completeness, nonetheless, let us recapitulate the features of monitoring which is usually accompanied by some classical and/or quantum noise (back-action). We characterize this back-action by a further diffusion constant D_{m} . The complete Liouville equation (4) reads:

$$\frac{d\rho}{dt} = \{H, \rho\} + \eta \frac{\partial}{\partial p} p \rho + (D_{\text{sp}} + D_{\text{th}} + D_{\text{m}}) \frac{\partial^2}{\partial p^2} \rho. \quad (9)$$

Suppose we start to measure the temperature of the oscillator at $t = 0$. The initial state of the oscillator is the Gibbs state (5) of temperature $T + \Delta T_{\text{sp}}$. When the ‘thermometer’ is switched on, the measurement noise starts to heat the oscillator towards the new stationary Gibbs state of temperature increased by

$$\Delta T_{\text{m}} = \frac{D_{\text{m}}}{m k_B} \tau. \quad (10)$$

Trivial dynamics of heating follows from Eq. (9) in the limit $\eta \ll \Omega$:

$$T'(t) = T + \Delta T_{\text{sp}} + (1 - e^{-t/\tau}) \Delta T_{\text{m}}. \quad (11)$$

Observe that the temperature effect of back-action is gradually reaching its steady state value. Back-action can be ignored for times much shorter than $\Delta T_{\text{m}}/\Delta T_{\text{sp}}$ times τ .

There is no fundamental limitation on the measurement precision (fluctuations) δT_{m} in the classical domain. There is a quantum tradeoff between the spectral components of δT_{m} and ΔT_{m} at a chosen frequency ω :

$$\delta T_{\text{m}} \Delta T_{\text{m}} \geq \frac{\hbar^2}{4k_B^2} \frac{|\Omega^2 - \omega^2 + i\eta\omega/2|^2}{\eta^2}, \quad (12)$$

as it follows from Refs. [20], cf. also Ref. [18]. The minimum of $\delta T_{\text{m}} + \Delta T_{\text{m}}$ is achieved when

$$\delta T_{\text{m}} = \Delta T_{\text{m}} = \frac{\hbar}{2k_B} \frac{|\Omega^2 - \omega^2 + i\eta\omega/2|}{\eta} \equiv \Delta T_{\text{SQL}} \quad (13)$$

which is called the standard quantum limit. This limitation concerns the steady state spectral component of the precision and back-action, respectively. For monitoring duration much shorter than τ (yet sufficient to gather significant data on ΔT_{sp}) the back-action won’t influence the system, we can choose finer precisions δT_{m} than ΔT_{SQL} .

	10^2	10^3	10^4	10^5	10^6
10^5 Hz	$[10^{-8} \text{ K}]$	$[10^{-7} \text{ K}]$	$[10^{-6} \text{ K}]$	10^{-5} K	10^{-4} K
10^4 Hz	$[10^{-7} \text{ K}]$	10^{-6} K	10^{-5} K	10^{-4} K	10^{-3} K
10^3 Hz	10^{-6} K	10^{-5} K	10^{-4} K	10^{-3} K	10^{-2} K
10^2 Hz	10^{-5} K	10^{-4} K	10^{-3} K	10^{-2} K	10^{-1} K
10 Hz	10^{-4} K	10^{-3} K	10^{-2} K	10^{-1} K	1 K
1 Hz	10^{-3} K	10^{-2} K	10^{-1} K	1 K	10 K

TABLE I: Magnitudes of spontaneous heating effect ΔT_{DP} of the DP-model on classical oscillators are shown at currently available or nearly available combinations of frequencies Ω and quality factors Q . The spatial resolution $\sigma_{\text{DP}} = 10^{-12}$ cm assumes the strongest effect, lattice constant is set to 500 pm. Data around the upper-left corner (it brackets) are not in the classical domain $k_B \Delta T_{\text{DP}} \gg \hbar \Omega$. Data above the millikelvin range are enhanced (typed in boldface) because their detection may not request millikelvin cooling or cooling at all.

Spontaneous heating: DP-model. In the gravity-related spontaneous collapse model (DP-model), the spontaneous diffusion is proportional to the Newton constant G . For the simple example of oscillator mass considered in [18]:

$$D_{\text{DP}} = \frac{\hbar}{2} m \omega_G^2 = \frac{\hbar}{2} m \frac{4\pi G \rho}{3} \left(\frac{a}{2\sqrt{\pi} \sigma_{\text{DP}}}\right)^3 \quad (14)$$

System	m	$\Omega/2\pi$ (Hz)	Q	T (K)	ΔT_{DP} (K)
gravitational wave detector [22]	40 kg	1	25000	300	0.16
suspended disc [19]	5 mg	0.5	5×10^5	300	6.4
SiN membrane [23]	34 ng	1.6×10^6	1100	4.9	$[4.4 \times 10^{-9}]$
aluminium membrane [24]	48 pg	1.1×10^7	3.3×10^5	0.015	$[1.9 \times 10^{-7}]$

TABLE II: Spontaneous heating ΔT_{DP} for the selection of opto-mechanical setups quoted in [18]. Values ΔT_{DP} are calculated from Eq. (16), assuming the largest spontaneous decoherence rates considered for the time being, corresponding to $\omega_G = 1.3\text{kHz}$. Two of the data (in brackets) are not in the classical domain $k_B \Delta T_{\text{DP}} \gg \hbar\Omega$.

where ρ is the mass density, and a is the lattice constant, while ω_G is the effective parameter used by [3, 4]. The spatial resolution σ_{DP} is the free parameter of the DP-model, conjectured to be in the following range [3]:

$$10^{-12}\text{cm} \lesssim \sigma_{\text{DP}} \lesssim 10^{-5}\text{cm}. \quad (15)$$

The expression (14) is valid for $\sigma_{\text{DP}} \ll a$. In this range, D_{DP} is independent of the shape of the mass while it depends on its microscopic structure. Using (14) for D_{sp} , we can write (7) as

$$\Delta T_{\text{DP}} = \frac{\hbar\omega_G^2}{2k_B}\tau, \quad (16)$$

where ω_G^2 is read out from (14). It is remarkable that ΔT_{DP} does not depend on the mass m .

Now we assume the strongest possible DP-decoherence, i.e., we take the finest conjectured spatial resolution $\sigma_{\text{DP}} = 10^{-12}\text{cm}$, also favored by particular arguments [3, 4]. If the lattice constant is set to $a = 5 \times 10^{-8}\text{cm}$, for concreteness, we obtain $\omega_G \approx 1.3\text{kHz}$ for the effective parameter. The spontaneous heating effect (16) can be written as

$$\Delta T_{\text{DP}} \approx \tau[s] \times 4.0 \times 10^{-5}\text{K}. \quad (17)$$

This is a convenient expression of the effect ΔT_{DP} to discuss possible choices of the frequency Ω and the quality factor $Q = \Omega\tau$ of the oscillator. The mass m has, as we noticed before, canceled from ΔT_{DP} .

Experimental implications. Applying Eq. (17) to a broad range of frequencies Ω and quality factors Q , we calculated the spontaneous heating ΔT_{DP} in Table I.

The lesson is transparent. If $\Delta T_{\text{DP}} \gg \hbar\Omega/k_B$, and this is the case except for a few highest Ω and lowest Q examples (in brackets), the DP-effect would prevent us from ground state cooling. This should be a significant detectable effect. But we do not need to try ground state cooling, the heating effect ΔT_{DP} equally shows up

far from the ground state. Low frequency oscillators with high quality factors are the favorable testbed. If the ring-down time $\tau = Q/\Omega$ of the oscillator is chosen between 10^2s and 10^6s , the spontaneous heating ΔT_{DP} scales between 1 mK and 10 K, respectively. This is a striking result. It is clear that classical (non-quantum) ‘thermometers’ of precision $\delta T_{\text{m}} \sim 1\text{mK}$ should exist. Technically, nonetheless, we might need to operate the measurement device in the quantum domain especially when the oscillator itself cooled and/or controlled via high precision quantum devices. Even in this case the oscillator is assumed to stay away from its ground state since the effect ΔT_{DP} is robust classical.

Following Ref. [18], and for a selection of experiments considered therein, we calculated the effect ΔT_{DP} , see Table II. The experiments [22] and [19], both performed at room temperature $T = 300\text{K}$, might be the promising ones. On the one hand, cooling is a reserve of higher sensitivity of detecting ΔT_{DP} . On the other hand, the experiment [19] even at room temperature must be sensitive to the 6.4 K spontaneous warming up.

As we mentioned before, monitoring may be neither convenient nor sufficient for detection. Let us consider the constraint (12) at the detection band around $\omega = 2\pi \times 500$, yielding $\delta T_{\text{m}}\Delta T_{\text{m}} = \Delta T_{\text{SQL}}^2 = (37\text{K})^2$. Such a standard quantum limit 37 K gives insufficient precision on the steady state, i.e.: in monitoring of duration much longer than $\tau = 1.6 \times 10^5\text{s}$. If we choose $\delta T_{\text{m}} = 1\text{K}$ the duration of monitoring must be limited to the order of hundred seconds before the back-action reaches the range of 1 K. This is obviously not the way to go in general. In this particular experiment measurement precisions below 1 K are not available by standard quantum monitoring. A single-pulse measurement must be considered instead, where state preparation is followed by a single one-shot measurement and the preparation-detection cycle is repeated many times.

CSL-model. In the CSL model the diffusion constant is proportional to the rate parameter λ_{CSL} . For the per-

pendicular momentum diffusion of a disk of thickness d

it reads

$$D_{\text{CSL}} = \lambda_{\text{CSL}} \frac{\hbar^2}{m_0^2} 4\pi\sigma_{\text{CSL}}^2 \frac{\varrho m}{d}, \quad (18)$$

where m_0 is the standard atomic unit. The value of the CSL collapse rate parameter has been constrained by a lower [6] and an upper estimate [7], cf. also [1]:

$$2.2 \times 10^{-17} \text{ Hz} \lesssim \lambda_{\text{CSL}} \lesssim 2.2 \times 10^{-8 \pm 2} \text{ Hz}. \quad (19)$$

Using D_{CSL} (18) for D_{sp} in (7) yields

$$\Delta T_{\text{CSL}} = \lambda_{\text{CSL}} \frac{\hbar^2}{m_0^2 k_B} 4\pi\sigma_{\text{CSL}}^2 \frac{\varrho}{d} \tau. \quad (20)$$

Note that the shape (thickness) of the oscillator matters, the mass m does not.

Suppose the strongest CSL decoherence rate from the range (19), let's take the estimate $\lambda_{\text{CSL}} = 2.2 \times 10^{-8 \pm 2} \text{ Hz}$ [7]. Using this value in (20) we obtain

$$\Delta T_{\text{CSL}} \approx \tau[s] \frac{\varrho[\text{g/cm}^3]}{d[\text{cm}]} \times 3.2 \times 10^{-6 \pm 2} \text{ K}. \quad (21)$$

Recall that $d \gg \sigma_{\text{CSL}} = 10^{-5} \text{ cm}$, hence the strongest heating effect is achieved when $d \approx \sigma_{\text{CSL}}$, leading to

$$\Delta T_{\text{CSL}} \approx \tau[s] \times 6.2 \times 10^{-1 \pm 2} \text{ K}, \quad (22)$$

where we kept $\varrho = 2 \text{ g/cm}^3$ as before. Comparing this result with (17) we conclude that, in classical oscillators, the strongest conjectured CSL effect ΔT_{CSL} would exceed the strongest conjectured DP effect ΔT_{DP} by at least two orders of magnitude.

Let us consider the $\Omega = 3.14 \text{ Hz}$ oscillator [19], also discussed in Ref. [18] in the context of the CSL model. Recall that the strongest DP-effect turned out to be $\Delta T_{\text{DP}} = 6.4 \text{ K}$, cf. Table I. This oscillator has the high quality factor $Q = 5 \times 10^5$, the ring-down time is extreme long: $\tau = 1.6 \times 10^5 \text{ s}$. The resonator is a 5 mg disk of thickness $d = 0.2 \text{ mm}$, Eq. (21) yields the spontaneous heating $\Delta T_{\text{CSL}} = 5.1 \times 10^{1 \pm 2} \text{ K}$, corresponding to the rates $\lambda_{\text{CSL}} = 2.2 \times 10^{-8 \pm 2}$, respectively. Obviously the values $\lambda_{\text{CSL}} \gtrsim 10^{-8}$ are not compatible with the experiment and the values $\lambda_{\text{CSL}} \sim (10^{-9} - 10^{-10})$ remain to be challenged.

Summary. The so far hypothetical spontaneous wavefunction collapse on massive degrees of freedom possesses a complementary classical effect: classical momentum diffusion. This produces a certain spontaneous increase ΔT_{sp} of the equilibrium temperature. This typical classical effect must be testable classically, without facing the standard quantum limitations of sensing. Therefore we must get spontaneous diffusion in the cross hairs instead of spontaneous collapse. We have derived the spontaneous heating ΔT_{sp} for mechanical oscillators in

classical thermal state, only using the classical Einstein-Smoluchowski relation, and found that ΔT_{sp} is proportional to the relaxation (ring-down) time, is independent of the mass. Experimental implications become transparent for both leading models DP and CSL of spontaneous collapse. We conclude that currently available extreme low-loss mechanical oscillators can already confirm the presence of spontaneous diffusion if its rate is close to the conjectured maximum. Alternatively, they enforce the update of the current constraints, cf. in Refs. [1, 21], on collapse model's parameters. The requested measurement precisions 1 mK - 1 K may not be reached in standard steady state quantum monitoring. We suggested that state tomography will fit the demands.

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